![EPFL]

Hydropower Plants: Generating and Pumping Units
Mock exam – part 2

Duration : 2 hours;

Documentation: personal hand written notes, bilingual dictionary, and available lectures and exercises on the Moodle are authorized; you can use your laptop in offline mode (no internet connection).

Exam evaluation: The weight of each question is indicated. General presentation and clarity of answers will be taken into account for the evaluation.

Maximum total score: 30 points

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1 SPECIFIC ENERGY LOSS CALCULATION

The schematic of a hydropower plant is shown in Figure 1. The reservoir level changes throughout the year due to seasonal effect. The maximum and minimum annual headwater levels are $Z_B^{\max}$ and $Z_B^{\min}$, respectively. Similarly, the tailwater level ranges between $Z_{\bar{B}}^{\max}$ and $Z_{\bar{B}}^{\min}$. The headwater and tailwater levels are maximum during summer and minimum in winter. <u>The power plant has 4 turbines</u>. Answer to the questions based on the values provided in Table 1.
The gravity acceleration, the water density and the water kinematic viscosity are:

$$g = 9.81 \text{ m s}^{-2},\ \rho = 998 \text{kg} \cdot \text{m}^{-3} \text{ and } \nu_{water} = 10^{-6} \text{ m}^2 \text{ s}^{-1}$$

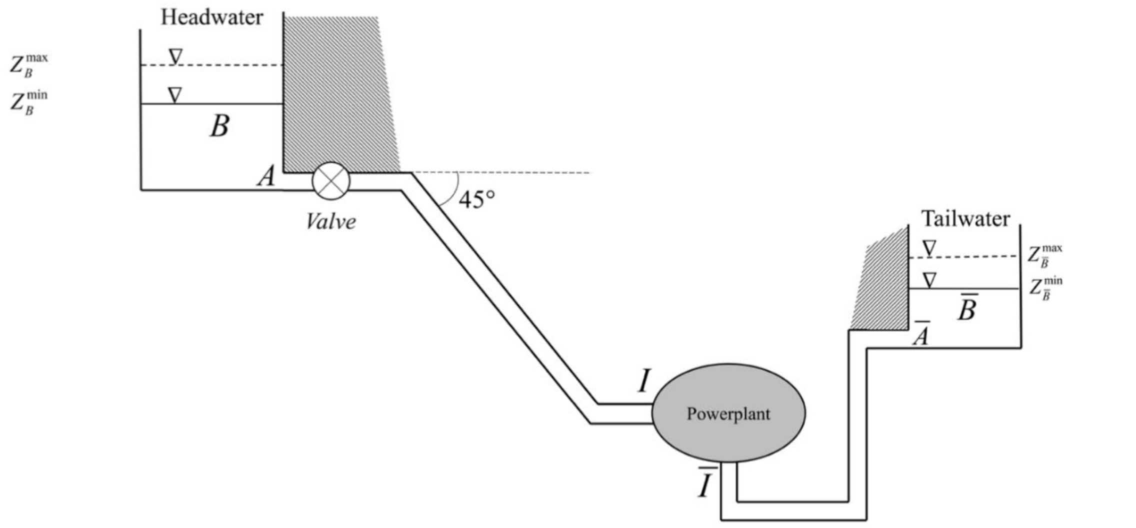


Figure 1: Simplified layout of the power plant.

Data	*Symbol*	*Value*	*Unit*
Max headwater reservoir level	$Z_B^{\max}$	780	m
Min headwater reservoir level	$Z_B^{\min}$	769	m
Max tailwater reservoir level	$Z_{\bar{B}}^{\max}$	575	m
Min tailwater reservoir level	$Z_{\bar{B}}^{\min}$	572	m
Number of turbines in the power plant	$z_{turbines}$	4	-
Rated discharge per turbine	Q	55	m^3s^{-1}
Penstock diameter	D_{pen}	5	m
Penstock length	L_{pen}	180	m
Pipeline roughness	k_s	5×10^{-5}	m
Elbow loss coefficient	k_{elbow}	0.15	-
Intake loss coefficient	k_{intake}	1.00	-
Valve loss coefficient	k_{valve}	0.10	-
Energetic efficiency	η_e	0.92	-
Volumetric efficiency	η_q	0.99	-
Global machine efficiency	η	0.90	-
Number of pole pairs	$z_{p,pairs}$	8	-
Frequency of the grid	f_{grid}	50	Hz
Turbine inlet section diameter	D_{1e}	3.50	m
Turbine inlet section height	B	0.60	m
Turbine outlet section diameter	$D_{\bar{1}e}$	2.80	m

Table 1: Technical data of the power plant.

Do not consider, for now, the season at which the power plant is operating.

1) Express the potential specific energy of the installation by Z_B, $Z_{\bar{B}}$, and *g*. [1 points]

   The potential specific energy of the installation is $g(Z_B - Z_{\bar{B}})$

2) Taking into account the specific energy losses $gH_{rB\div I}$ and $gH_{r\bar{I}\div\bar{B}}$, express the available specific energy *E* by *g*, Z_B, $Z_{\bar{B}}$, $gH_{rB\div I}$ and $gH_{r\bar{I}\div\bar{B}}$. [1 points]

   The available specific energy can be written as $E = g(Z_B - Z_{\bar{B}}) - gH_{rB\div I} - gH_{r\bar{I}\div\bar{B}}$

3) Calculate the water velocity in the penstock. [1.5 points]

The penstock brings the water to the power plant, where 4 turbines are generating power. To calculate the water velocity in the penstock we need to consider the entire amount of the power plant discharge: $Q_{penstock} = z_{turbines} \cdot Q$

The water velocity then reads:

$$C_{pen} = \frac{z_{turbines} \cdot Q}{\pi \left(\frac{D_{pen}}{2} \right)^2} = 11.2\,\mathrm{m\,s^{-1}}$$

4) Calculate the Reynolds number in the penstock. [1 points]

The Reynolds number in the penstock is:

$$\mathrm{Re}_{pen} = \frac{C_{pen} \cdot D_{pen}}{\nu_{water}} = 5.6 \times 10^7$$

5) Calculate the sum of the singular and distributed energy losses along the penstock, $gH_{rB \div I}$, for the power plant discharge. [3 points]

To calculate the distributed losses term λ *in* $gH_{r_{B\div I},distr} = \lambda \frac{L_{pen}}{D_{pen}} \frac{C_{pen}{}^2}{2}$, *it is necessary to compute the local loss coefficient with Churchill's formula for the power plant discharge. To do so, we use the penstock velocity and Reynolds number computed in questions 3) and 4).*

$$A = \left[2.457 \times \ln \frac{1}{\left(\frac{7}{Re_{pen}} \right)^{0.9} + 0.27 \frac{k_s}{D_{pen}}} \right]^{16} = 7.28 \cdot 10^{23}$$

$$B = \left[\frac{37530}{Re_{pen}} \right]^{16} = 1.66 \cdot 10^{-51}$$

Which yields:

$$\lambda = 8 \cdot \left[\left(\frac{8}{\mathrm{Re}_{pen}} \right)^{12} + \frac{1}{(A+B)^{\frac{3}{2}}} \right]^{\frac{1}{12}} = 8.3 \cdot 10^{-3}$$

Using the definition of the singular losses, it is possible to express all the losses in a single, considering the intake, the valve and the two elbows or the singular losses:

$$gH_{r_{B\div I}} = \sum gH_{r_{B\div I},sing} + gH_{r_{B\div I},distr}$$
$$= \left(k_{intake} + k_{valve} + k_{elbow} + k_{elbow} + \lambda \frac{L_{pen}}{D_{pen}} \right) \frac{C_{pen}{}^2}{2}$$

Using the losses coefficients listed in Table 1 and the values computed previously:

$$
\begin{aligned}
gH_{r_{B\div I}} &= \sum gH_{r_{B\div I},sing} + gH_{r_{B\div I},reg} \\
&= \left(k_{intake} + k_{valve} + k_{elbow} + k_{elbow} + \lambda \frac{L_{pen}}{D_{pen}} \right) \frac{C_{pen}{}^2}{2} \\
&= \left(1 + 0.1 + 2 \cdot 0.15 + 8.3 \times 10^{-3} \cdot \frac{180}{5} \right) \cdot \frac{11.2^2}{2} = 106.55 \ J \cdot kg^{-1}
\end{aligned}
$$

For the following questions, consider that the power plant operates during a day in the summer season.

6) Calculate the available specific energy and the hydraulic power of one turbine. The specific energy losses in the tailrace channel $gH_{r_{\bar{I}\div\bar{B}}}$ is assumed to be 0.1% of the potential specific energy. [2 points]

The available specific energy can be written as:

$$E = g(Z_B - Z_{\bar{B}}) - gH_{rB\div I} - gH_{r\bar{I}\div\bar{B}} = 9.81 \cdot (780 - 575) \cdot (1 - 0.001) - 106.55 = 1'902.49 \ J \cdot kg^{-1}$$

The hydraulic power of one turbine can be written as:

$$P_h = \rho Q E = 998 \cdot 55 \cdot 1'902.49 = 104.43 \ MW$$

7) Calculate the transferred power and the output power of the turbine. [2 points]

The transferred power can be written as:

$$P_t = \eta_e \eta_q P_h = 0.92 \cdot 0.99 \cdot 104.43 = 95.12 \ MW$$

The output power reads:

$$P = \eta P_h = 0.90 \cdot 104.43 = 93.99 \ MW$$

The energy losses and the leakage flow losses should be accounted for when computing the transferred power, which thus depends on the energetic and volumetric efficiencies. The output power is only computed with the global machine efficiency, as it already accounts for the volumetric efficiency.

2 POWER TRANSFER IN THE HYDRAULIC MACHINE

The power plant studied so far features four generating units, each one equipped with a Francis turbine. Let's now have a closer look to one of those turbines, whose meridional view is shown in Figure 2.

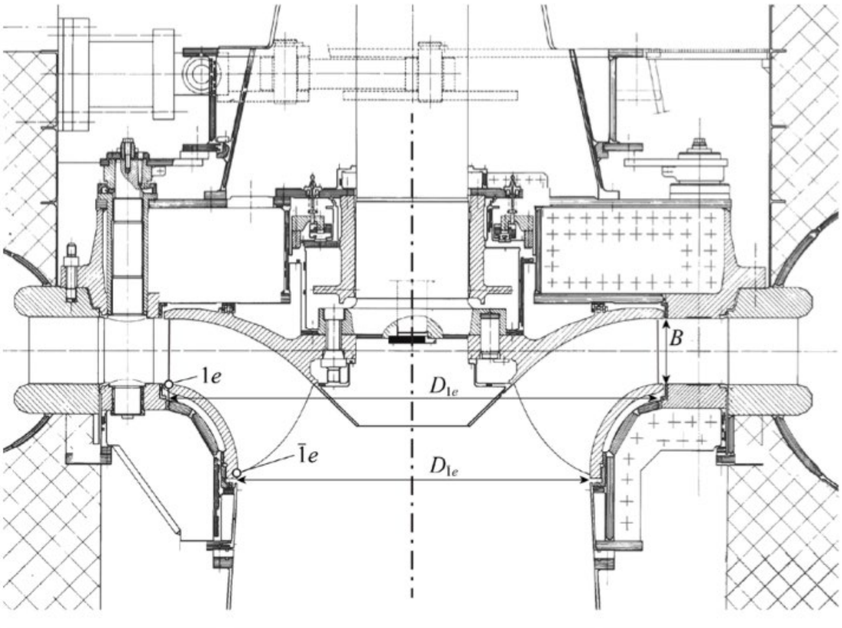


Figure 2: Meridional view of the Francis turbine.

8) Express and compute the external runner rotating velocity at the runner inlet and outlet, respectively U_{1e} and $U_{\bar{1}e}$. [2 points]

To compute the runner rotating velocity components, let's compute ω first:

$$\omega = 2\pi n = 2\pi \frac{f_{grid}}{z_{p,pairs}} = 39.27 \ rad \cdot s^{-1}$$

The external rotating velocities are computed with the inlet and outlet external diameters:

$$U_{1e} = \omega \frac{D_{1e}}{2} = 39.27 \cdot \frac{3.5}{2} = 68.72 \ m \cdot s^{-1}$$

$$U_{\bar{1}e} = \omega \frac{D_{\bar{1}e}}{2} = 39.27 \cdot \frac{2.8}{2} = 54.98 \ m \cdot s^{-1}$$

9) Express and compute the inlet and outlet surface sections of the turbine A_{1e} and $A_{\bar{1}e}$. [2 points]

The inlet surface is computed as:

$$A_{1e} = D_{1e} \cdot \pi \cdot B = 3.5 \cdot \pi \cdot 0.6 = 6.60 \ m^2$$

And the outlet surface reads as:

$$A_{\bar{1}e} = \frac{D_{\bar{1}e}^{\ 2}}{4} \cdot \pi = \frac{2.8^2}{4} \cdot \pi = 6.16 \ m^2$$

10) Express the meridional component of the absolute flow velocity at the runner inlet and outlet, respectively Cm_{1e} and $Cm_{\bar{1}e}$ as a function of the discharge Q, the volumetric efficiency η_q and the sections A_{1e} and $A_{\bar{1}e}$. [2 points]

The meridional components are expressed as follows:

$$Cm_{1e} = \frac{Q \cdot \eta_q}{A_{1e}}$$

$$Cm_{\bar{1}e} = \frac{Q \cdot \eta_q}{A_{\bar{1}e}}$$

11) Express the tangential component of the absolute flow velocity at the runner inlet and outlet, respectively Cu_{1e} and $Cu_{\bar{1}e}$, as a function of the absolute velocity angles α_{1e} and $\alpha_{\bar{1}e}$, and of the meridional components Cm_{1e} and $Cm_{\bar{1}e}$ from the previous question. [2 points]

The tangential components are computed using trigonometry as follows:

$$Cu_{1e} = \frac{Cm_{1e}}{\tan\left(\alpha_{1e}\right)}$$

$$Cu_{\bar{1}e} = \frac{Cm_{\bar{1}e}}{\tan\left(\alpha_{\bar{1}e}\right)}$$

Consider now the operating condition studied in questions 6) and 7), i.e. a summer day with the rated discharge value for each turbine, and assume that this condition is the turbine BEP.

12) Compute the meridional component of the inlet and outlet absolute velocities. [2 points]

The meridional components are computed as follows:

$$Cm_{1e} = \frac{Q \cdot \eta_q}{A_{1e}} = \frac{55 \cdot 0.99}{6.6} = 8.25 \ m \cdot s^{-1}$$

$$Cm_{\bar{1}e} = \frac{Q \cdot \eta_q}{A_{\bar{1}e}} = \frac{55 \cdot 0.99}{6.2} = 8.78 \ m \cdot s^{-1}$$

13) Compute the inlet absolute and relative flow angle α_{1e} and β_{1e}. [2.5 points]

At BEP, the outflow is purely axial. This means that the outlet tangential component of the absolute flow is $Cu_{\bar{1}e} = 0$, and the Euler equation reduces to $E_t = Cu_{1e} \cdot U_{1e}$.

Using the value of transferred power computed in question 7), the transferred specific energy can be deduced:

$$E_t = \frac{P_t}{\rho Q \eta_q} = \frac{95.12MW}{998 \cdot 55 \cdot 0.99} = 1750.43 \ J \cdot kg^{-1}$$

The inlet angles are therefore computed as follows:

$$\alpha_{1e} = \tan^{-1}\left(\frac{Cm_{1e}}{Cu_{1e}}\right) = \tan^{-1}\left(\frac{Cm_{1e}}{E_t / U_{1e}}\right) = \tan^{-1}\left(\frac{8.25}{1750.43/68.72}\right) = 17.95^\circ$$

$$\beta_{1e} = \tan^{-1}\left(\frac{Cm_{1e}}{U_{1e} - Cu_{1e}}\right) = \tan^{-1}\left(\frac{Cm_{1e}}{U_{1e} - E_t / U_{1e}}\right) = \tan^{-1}\left(\frac{8.25}{68.72 - \frac{1750.43}{68.72}}\right) = 10.80^\circ$$

14) Compute the outlet absolute and relative flow angle $\alpha_{\bar{1}e}$ and $\beta_{\bar{1}e}$. [2 points]

At BEP, the outflow is purely axial, meaning that $\alpha_{\bar{1}e} = 90^\circ$

The relative flow angle can be computed as:

$$\beta_{\bar{1}e} = \tan^{-1}\left(\frac{Cm_{\bar{1}e}}{U_{\bar{1}e}}\right) = \tan^{-1}\left(\frac{Cm_{\bar{1}e}}{U_{\bar{1}e}}\right) = \tan^{-1}\left(\frac{8.78}{56.94}\right) = 8.77^\circ$$

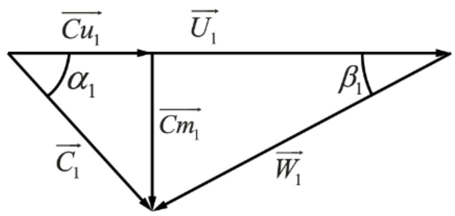
15) Compute the norm of the inlet and outlet relative flow velocities $\left|\vec{W}_{1e}\right|$ and $\left|\vec{W}_{\bar{1}e}\right|$. [2 points]

The norm of the inlet and outlet relative flow can be computed as:

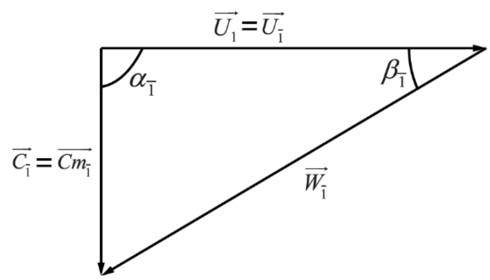
$$\left|W_{1e}\right| = \frac{U_{1e} - Cu_{1e}}{\cos(\beta_{1e})} = \frac{68.72 - \frac{1750.43}{68.72}}{\cos(10.80)} = 44.03 \ m \cdot s^{-1}$$

$$\left|W_{\bar{1}e}\right| = \frac{U_{\bar{1}e}}{\cos(\beta_{\bar{1}e})} = \frac{56.94}{\cos(8.77)} = 57.61 \ m \cdot s^{-1}$$

16) Sketch qualitatively the inlet and outlet velocity triangles for this operating condition. [2 points]



Velocity triangle at the runner inlet



Velocity triangle at the runner outlet